Name:_____ Date:_____

SOL M12P HW Section 5.2 Graphing Sine and Cosine Function With Transformations

$$y = Asin(x - C) + D \quad and \quad y = Acos(x - C) + D$$

1. How does the constant "A" transform the trigonometric functions above? Explain:
   The constant "A" gives the vertical expansion/compression. If "A" >1 → we get a vertical expansion.
   If "A" is between 0 and 1 → we get a vertical compression. If "A" is negative, we have a vertical reflection.
   Generally, the constant "A" is the amplitude.

2. How does the constant "C" transform the trigonometric functions above? Explain:
   The constant "C" gives the horizontal shift. The same rules from CH1 "Transformations" also applies here. The constant "C" also tells us where the function begins.

3. How does the constant "D" transform the trigonometric functions above? Explain:
   The constant "D' gives the vertical shift. If "D" is positive, the graph shifts up. If "D" is negative, the graphs shifts down.

4. What is the range of a sine or cosine function in terms of "A" , "C", and "D"? Explain:

   The highest points of a sine or cosine graph is D +|A|. The lowest points of a sine or cosine graph is D – |A|.
   So the range is from "D - |A|" to "D + |A|".

5. What is the value of "B" in the function $y = AsinB(x - C) + D$ for y= $4sin(x - 2) + 3$? How does the constant "B" transform the sine function? Explain:
   The value of "B" in this equation is 1, since there isn't a number in front of "x". The value of tells us what the period is. $Period = \frac{2\pi}{B}$

6. Does the function $y = 3sin(x - 4) + 5$ have any X-intercepts? Explain:

   The graph is shifted 5 units above the "x" axis and has an amplitude of 3. This means that the lowest points of the graph will be 2 units above the 'x' axis. Therefore, it will not have any x-intercepts.

7. How would you tell if a sine or cosine function has any X-intercepts by looking at their equations? Explain:

   If $|A| > |D|$, then the amplitude is greater than the shift. Therefore, we will have "x" intercepts. In contrast, if $|A| < |D|$, then the amplitude is less than the vertical shift. Therefore, we will have no "x" intercepts.
   Just to make sure, $|A|$ is the absolute value of "A".

8. What are the x-intercepts of the function: $y = 3sin\left(x - \frac{\pi}{3}\right) + 2$. Provide a general formula for all the X-intercepts.

To find the "x" intercepts, first make 'y' equal to zero. Then use algebra to solve for "x".

$0 = 3\sin\left(\theta - \frac{\pi}{4}\right) + 0.5$

$\frac{-0.5}{3} = \sin\left(\theta - \frac{\pi}{4}\right)$

$\sin^{-1}\left(\frac{-1}{6}\right) = A$

$-0.167448,\ \ 3.30904 = A$

→

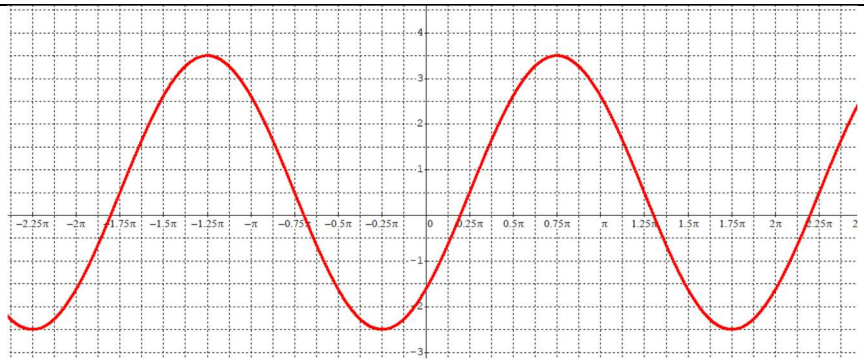
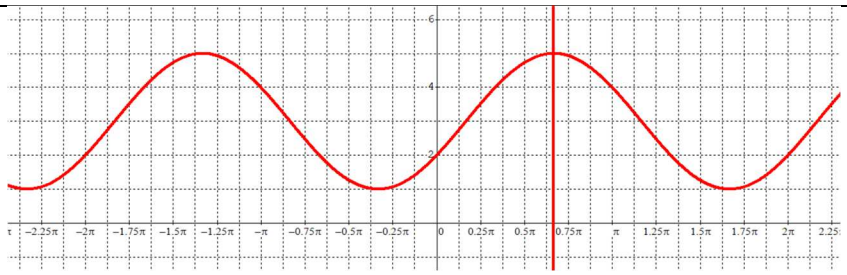
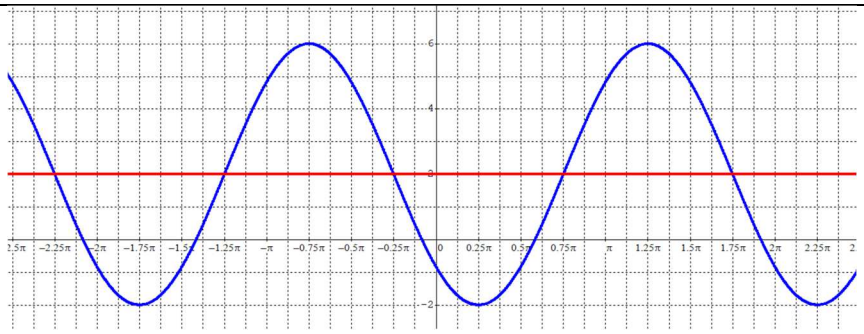
$-0.167448 = A,\ \ 3.30904 = A$

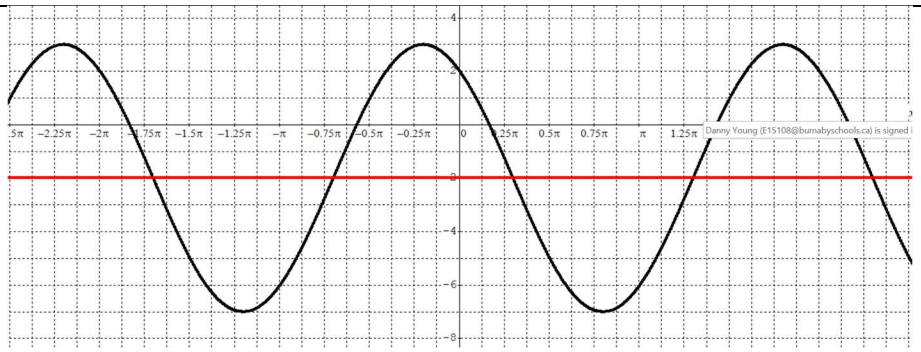
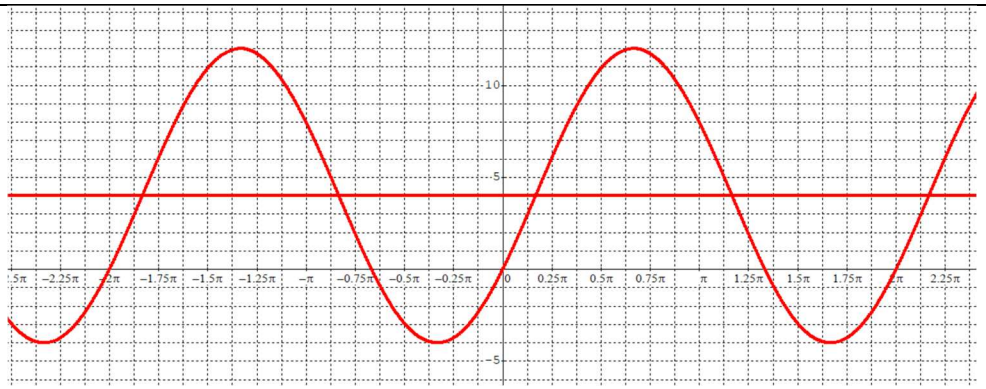
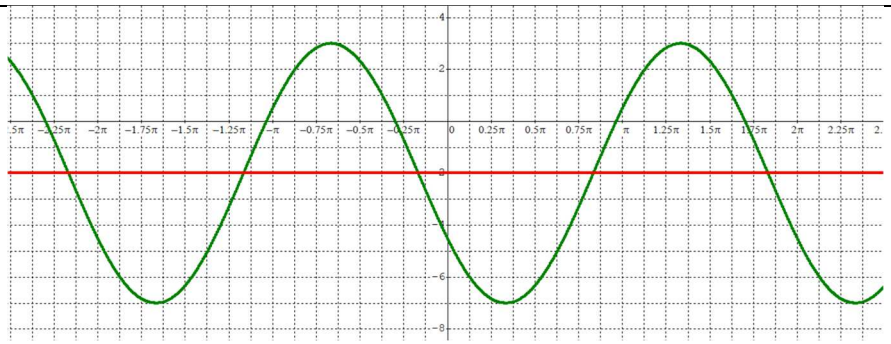
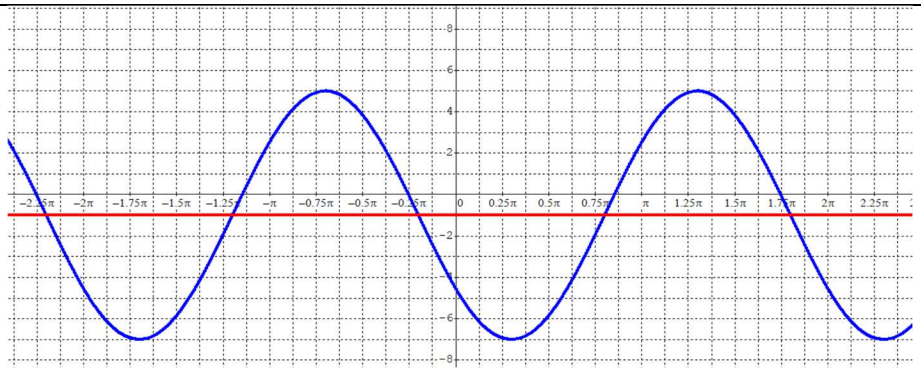
$-0.167448 = \theta - \frac{\pi}{4},\ \ 3.30904 = \theta - \frac{\pi}{4}$

$-0.167448 + \frac{\pi}{4} = \theta,\ \ 3.30904 + \frac{\pi}{4} = \theta$

$0.6179501^R, 4.0944^R = \theta$

9. For each of the following equations, find the constants "A" , "C" and "D". Then indicate the transformations involved. State the period, amplitude, domain, and range: Graph the function.

$y = 3\sin\left(\theta - \frac{\pi}{4}\right) + 0.5$ A: = 3 Amplitude =3 C: = pi/4 Shift of pi/4 to the right D = 0.5 Period : 2pi Domain : Range : $x \in \mathbb{R},\quad -2.5 \le y \le 3.5$	
$y = 2\cos\left(\theta - \frac{2\pi}{3}\right) + 3$ A: 2 Amplitude =2 C: 2pi/3 (shifts to the right) D: =3 (3 units up) Period : 2pi Domain : Range : $x \in \mathbb{R},\quad 1 \le y \le 5$	
$y = -4\sin\left(\theta + \frac{\pi}{4}\right) + 2$ A: - 4 Amplitude =4 C: pi/4 (shifted to the left) D:=2 Period : 2 pi Domain : Range : $x \in \mathbb{R},\quad -2 \le y \le 6$	

$y = 5\cos\left(\theta + \frac{\pi}{5}\right) - 2$ A: 5 Amplitude =5 C: pi/5 (shifted to the left) D: -2 (shifted 2 units down) Period : 2pi Domain : Range : $x \in \mathbb{R}$, $-7 \le y \le 3$	
$y = 8\sin\left(\theta - \frac{\pi}{6}\right) + 4$ A: 8 Amplitude =8 C: pi/6 (shifted to the right) D: 4 (shifted 4 units up) Period : 2pi Domain : Range : $x \in \mathbb{R}$, $-4 \le y \le 12$	
$y = -5\cos\left(\theta - \frac{\pi}{3}\right) - 2$ A: -5 Amplitude=5 C: pi/3 (shifted to the right) D:-2 Period : 2pi Domain : Range : $x \in \mathbb{R}$, $-7 \le y \le 3$	
$y = -6\sin\left(\theta + \frac{\pi}{5}\right) - 1$ A: -6 Amplitude=6 C: pi/5 (shifted to the left) D: -1 Period : 2pi Domain : Range : $x \in \mathbb{R}$, $-7 \le y \le 5$	

$y = 4.5\cos\left(\theta + \frac{\pi}{7}\right) - 3$

A: 4.5 Amplitude = 4.5

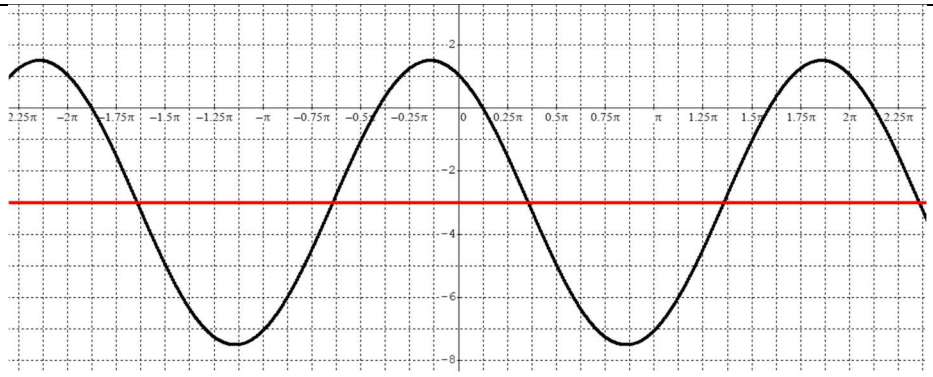
C: pi/7 (Shifted to the left)

D: = -3

Period : 2pi

Domain : $x \in \mathbb{R}$,

Range : $-7.5 \le y \le 1.5$



10. When setting the increments on the X-axis for graphing a trigonometric function, how do we determine the value of 1 increment on the grid? Ie: Suppose we are given the equation $y = A\sin\left(\theta - \frac{\pi}{3}\right) + D$, where the period is 2π and there is a shift of $\frac{\pi}{3}$to the right. How do we determine the value of one increment? Explain:

    Use the denominators of "C" or the period to determine the value of 1 increment. If we are shifting by $\frac{\pi}{3}$, then one increment should be a factor of $\frac{\pi}{3}$. Ie: we can make one increment equal to $\frac{\pi}{3}$ *or* $\frac{\pi}{6}$ or even perhaps $\frac{\pi}{12}$

11. What if we are given the equation $y = A\sin\left(\theta - \frac{\pi}{7}\right) + D$, where the period is 2π and there is a shift of $\frac{\pi}{7}$ to the right. How do we determine the value of one increment? Explain:

    Since we have a fraction over 7, we can make one increment as either $\frac{\pi}{7}$ *or* $\frac{\pi}{14}$

12. In order for the function $y = a\sin\left(\theta - c\right) + d$ to have an x-intercept, which of the following must be true?

    i) $a < d$ *ii)* $a + d = 1$ *iii)* $c < d$ *iv)* $d > a$ *v)* $a - d = 0$

In order to have an "x" intercept, then |a| must be greater than |d|. unfortunately, none of these work….sigh….oops…

13. Find the equation of a function in the form of $y = a\sin\left(x - b\right) + d$ with a maximum value of 3 when x=0 and a minimum of -2.

14. Find the amplitude and phase shift of the following function: $y = \sin^3\theta + \sin\theta\cos^2\theta$

    $y = \sin\theta\left(\sin^2\theta + \cos^2\theta\right)$

    Factor out the sine. $y = \sin\theta(1)$. So we end up with a sine function. Amplitude is 1 and phase

    $y = \sin\theta$

    shift is 0 units.

15. Find the equation of a function in the form of $y = a\cos b(x-c)+d$ with the following:

a. Maximum at 8, minimum at -2, phase shift of $\frac{\pi}{2}$ units to the right

b. Maximum point $\left(\frac{\pi}{3},10\right)$ next minimum point $\left(\frac{4\pi}{3},4\right)$